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Which Rod?
Units & the Addition of Fractions

Fractions: the overarching topic

"Among all the topics in K-12 curriculum, rational numbers arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, and the most essential to success in higher mathematics and science" (Lamon, 2007).

Topic 1: Unit Fraction

Unit fraction is a fraction with the numerator of 1. It shows how many pieces the unit is divided (partitioned) into. For example 1/7 shows that the unit is divided into 7 pieces. The same unit can be partitioned many different ways. The following diagram shows some possible partitioning of the same unit.

It might seem obvious, but it worth noticing that sizes of different unit fractions (even though parts of the same unit) are NOT the same. For example, 1/7 of a chocolate bar is smaller that 1/2 of the same chocolate bar. Later in the second activity of this vignette I use this concept.

Topic 2: The concept of "Addition of Fractions"

Conceptual understanding of the notion of "unit fraction" leads to better understanding of the addition of fractions. Students' questions, such as "*why don't we just add the tops (numerators) and then add the bottoms (denominators),"* show the lack of understanding of the "unit fraction". To properly solve an addition of fractions problem, students need to find a common denominator that is a common unit that fits in the denominators of both fractions.

What is important to point out is that adding fractions can be looked at as the continuation of adding whole numbers. We know that in adding whole numbers, the unit of counting is 1. For example, 7 is just 1 added to itself 7 times. Much the same way, every fraction is the repeated addition of a unit fraction. For example, 3/7 is the just the unit fraction of 1/7 added to itself 3 time.

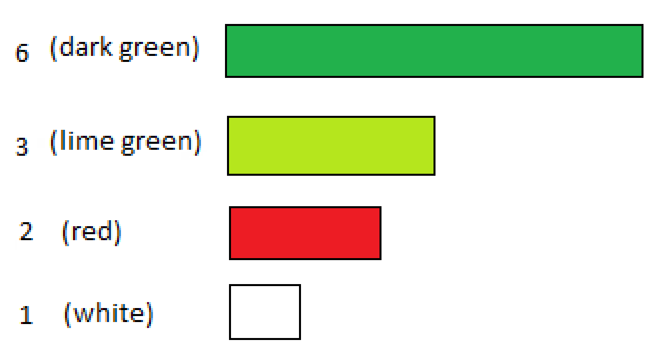
The attached three mathematical activities are designed for students to see why it is necessary to have a common unit when adding two pieces of different sizes (e.g. two Cuisenaire rods or two pieces of fraction circles).

Activity 1: which rods?

The objective of this activity is to re-emphasise the importance of a common unit in the concept of addition. The goal is for students to see that they cannot add (put together) pieces of different sizes to measure an object, without taking their sizes into account.

Material:

- 4 colours Cuisenaire rods – dark green, lime green, red, and white
- ribbons - 1 metre per group
- tape
- paper
- ruler
- pencil



- scissors

Task:

You are asked to make tails for 5 badges out of ribbon. You are given 4 colours Cuisenaire rods (dark green, lime green, red, and white) to size the ribbons. The manual said that each tail should be two rods long. Use the materials to make the tails. Tape each tail on your sheet and write its unit (i.e., two-rod long) in front of it. If everyone in you class follows the same steps, would all your badges look the same? Why? Does the unit of "two-rod long" show the differences? What better unit can you think of?

Description of the activity:

I used this activity with grade 5 students in Ottawa. This activity was designed as an introductory activity to emphasise the importance of having a common unit in solving the addition problems.

Step 1- Cutting the pieces

I teamed the children in groups of two or three and gave each team one set of Cuisenaire rods (the four colours mentioned above), ribbons, tape, paper, ruler, pencil, and scissors.

To be able to eventually connect this activity to the addition of fractions, I started with questions such as:

- How can I add two of these rods together? (A possible answer: putting together two segments end-to-end on a horizontal line)
- How can I cut pieces of my ribbons that are two rods long?

Children are allowed to use any combination of two rods to size and cut the pieces of ribbons… I asked them to be creative☺

Step 2- What is the unit?

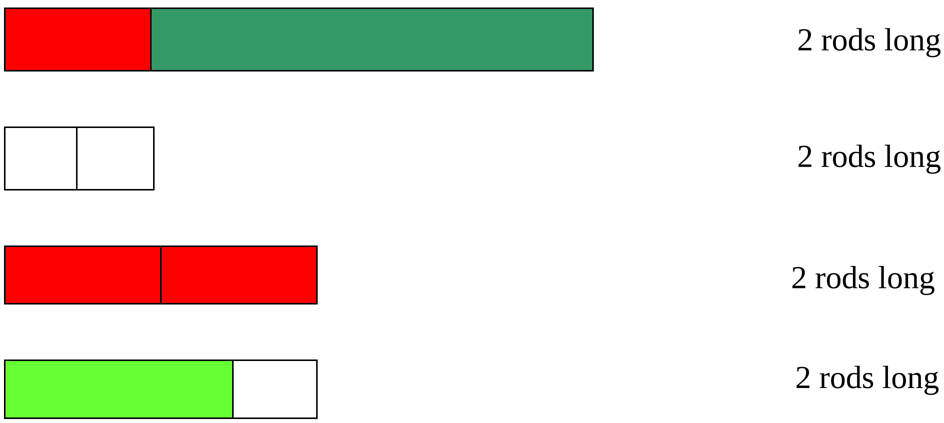
Then children were asked to tape the cut pieces of the ribbons on their paper and write the numerical value of each ribbon and its unit of measurement in front of each piece (i.e., 2 rods long).

The following prompting questions may generate some discussions:

- What did we use to size the ribbons? (rods).

- What do you think our unit of measurement would be?
- If I were to use paper clips…what would be my unit of measurement?
- How many rods does each piece of ribbon show? (Two)
- So how long is each of these pieces?
- How can I name them mathematically… with a number? (2 rods long)

The following diagrams show some of the possible combinations:



Step 3 – What is wrong?

Because students used different combinations of rods, their pieces of ribbon were not the same size. But they were all named “2 rods long”. What was wrong? To start the conversation, children were asked to bring their pieces of paper (with taped ribbons on it) and sit in a circle, so that they were able to see one another’s work.

The following questions may generate some discussion:

- Whose ribbon is 2 rods long? (Ask each team to show their ribbons to the group).
- Do you see any difference?
- Why do you think they are different? (Because different sizes of rods were used to measure and cut the ribbons)…
- So Kate’s 2 rods long ribbon may not be the same size as John’s? Yes?

Step 4 – How to address the issue?

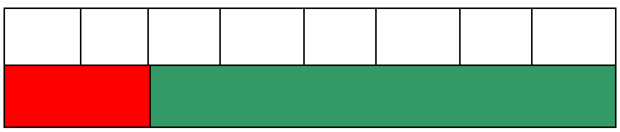
This step is designed for the students to think about the importance of common unit when adding. Children are encouraged to think about ways to name pieces of the ribbons according to their sizes. The objective is still to have the length as 2 rods long. So cutting pieces in a same size would not be a useful option.

The following questions may generate some discussion:

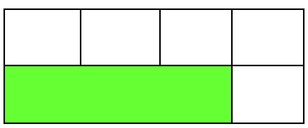
- How can we fix this?
- How can we give a new name to each piece so that the name shows the size?
- What can I use?

One possible answer is to use the white "unit" rod and see how many "units" will fit in each ribbon and call each piece according to the number of "units" that fit on it.

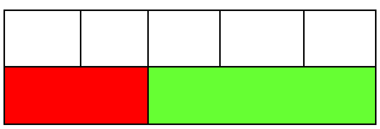
The following diagrams show this possible solution to the task. Now instead of calling all the pieces 2-rods long, pieces can be called 6 units long, 4 units long and so on.



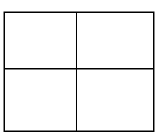
6 units long



4 units long



5 units long



2 units long

At the end of this activity, students maybe asked to write a few sentences or talk about the activity and the results as a group (i.e., this activity showed that one cannot add rods of different sizes, without taking their size into account, because the answer would not be representative of the sizes).

2- Transitional activity: Connection to the addition of fractions

To connect activity 1 to the concept of the addition of fractions, a new activity was designed. In this activity, I used the same 4 colours of rods – dark green, lime green, red, and white. However, this time I called the dark green a "unit" or "one". The goal of this activity is now to add (or put together) the red rod and lime rod considering they were representing fractional numbers.

The objective was to use the outcome of the first activity to conclude that a "common unit" (that is a rod that fits on both lime green and red rods) is needed to add the rods accurately.

To start the discussion the following questions may be asked:

- If the dark green rod is "one" what fraction would the lime green represent?
- What about the red or the white? (Lime green is 1/2, Red is 1/3, and White is 1/6).
- Now, if I want to cut pieces of ribbon using the red rod and lime green rod put together, what is the size of my ribbon?
- How can I give it a name that represents the size of the ribbon?
- Let's think of the previous activity?
- What did we do then to correctly name each ribbon?
- Can I do the same thing here? Why yes? Why no?

Students then are asked to see how many white rods would fit on both rods.

They need to remember that the white rod represents 1/6ths and five "whites" represent 5/6ths.

The following questions may generate some discussion:

- So how can I name this piece (lime and red rods put together) mathematically?
- What would be the number that shows its size?
- What is the size of the white rod?
- Remember that the green is 1. So what is the size of white? (1/6)
- How many white rods can we fit in a green? (6)
- How many white rods can I fit on the lime and red rods put together? (5)

- What is the size of the big rod (red and lime green together)? (5/6)

This transitional activity would lead to the second activity.

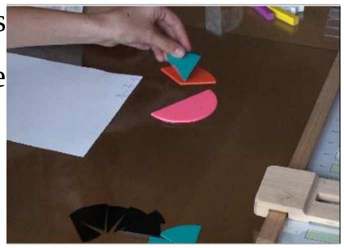
3- Activity 2: Fraction circles

The objective of this activity is for students to solve an addition of fractions problem using the fraction circles. The combination of the first activity and the transitional activity were designed to emphasise the importance of common unit while doing the addition of fractions problems. To extend children's understanding of the addition of fractions to a newer situation, students will be asked to solve an addition of fractions problem using the fraction circles (for example 1/2+ 2/5).

The following questions may guide the discussion:

- Are 1/2 and 2/5 the same size?
- Can I just add them up like that?
- What do I need to do?
- How can I make them show them with pieces of the same size?
- Which piece might help me?
- Which piece would fit on both pieces?

The pieces of fraction circle that fits in both the 1/2 and 2/5 is the 1/10 and students need 9 of the 1/10th pieces to cover the whole thing. That is 9/10.



So the answer to 1/2 + 2/5 = 9/10

References

Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Reston, VA: NCTM.